

Quiz 2

MATH 261 CALCULUS III, SPRING 2017

HOURL:

NAME:

Problem 1. Let r be the curve in \mathbb{R}^2 defined by

$$r(t) = \cos(2\pi t)\mathbf{i} + \sin(2\pi t)\mathbf{j} \quad \text{where } 0 \leq t \leq 1.$$

Provide the parametric equation for the line tangent to r at the following points:

1. $(0, 1)$
2. $(1, 0)$
3. $(0, -1)$
4. $(-1, 0)$.

Solution: To parametrize a line, we need an offset vector p_o and a direction vector v_o . Then the line is given by the parametrization

$$L(t) = p_o + tv_o$$

If r passes through the point (x_o, y_o) at time t_o , then the equation for the tangent line is then

$$L(t) = (x_o, y_o) + r'(t_o)t.$$

The curve passes through the given points at times $1/4, 0, -3/4$, and 1 (in order as given). Since $r'(t) = (-2\pi \sin(2\pi t), 2\pi \cos(2\pi t))$, the lines are given by

1. $L(t) = (0, 1) + t(-2\pi, 0)$
2. $L(t) = (1, 0) + t(0, 2\pi)$
3. $L(t) = (0, -1) + t(2\pi, 0)$
4. $L(t) = (-1, 0) + t(0, -2\pi)$.

Problem 2. Draw the curve r and the four tangent lines you computed in problem 1. Do all of this in one picture.

Solution: The picture is a circle of radius 1 centered at the origin, along with the lines $x = 1$, $x = -1$, $y = 1$, $y = -1$.