

Quiz 1

MATH 261, CALCULUS III, SPRING 2018

SECTION:

NAME: key

Instructions: Solve as many of these problems as you can. Circle the correct answer, and show your work!

Problem 1. Find the angle between the planes given by the equations

$$x + y = 2 \text{ and } x + y + \sqrt{2}z = \sqrt{6}$$

(a) $\pi/2$

(b) $\pi/4$

(c) $\pi/6$

(d) π

(e) $\pi/3$

Angle between planes = Angle between normal vecs.

$$\langle 1, 1, 0 \rangle \cdot \langle 1, 1, \sqrt{2} \rangle = |\langle 1, 1, 0 \rangle| |\langle 1, 1, \sqrt{2} \rangle| \cos \theta$$

$$1 + 1 = 2 = \sqrt{2} \cdot \sqrt{2} \cos \theta \Rightarrow \theta = \pi/4$$

Problem 2. Let $\vec{a} = (1, -1, 2)$ and $\vec{b} = (2, 1, 0)$. Find t such that the vector $\vec{c} = (5, t-1, 2)$ is perpendicular to $\vec{a} \times \vec{b}$.

(a) $t = 1$

(b) $t = 2$

(c) $t = -1$

(d) $t = -2$

(e) $t = 0$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 2 \\ 2 & 1 & 0 \end{vmatrix} = (-2, 4, 3)$$

$$\vec{c} \text{ is perp to } \vec{a} \times \vec{b} \Leftrightarrow \vec{c} \cdot (\vec{a} \times \vec{b}) = 0$$

$$\text{So } 5t - 4 = -8, t = -2$$

$$\begin{pmatrix} 5t-1 \\ 2 \end{pmatrix} \cdot (-2, 4, 3) = 0 \Rightarrow -10 + 4t + 6 = 4t - 4 = 0 \Rightarrow t = 1$$

Problem 3 The plane passing through $(0, 1, 0)$ and parallel to the plane $x + y - 2z = 3$ intersects the x axis at the point:

(a) $(-1, 0, 0)$

(b) $(1, 0, 0)$

(c) $(-2, 0, 0)$

(d) $(2, 0, 0)$

(e) $(-3, 0, 0)$

Parallel to $x + y - 2z = 3 \Leftrightarrow$ of form $x + y - 2z = c$

Passing thru $(0, 1, 0) \Leftrightarrow 0 + 1 - 2 \cdot 0 = c \Leftrightarrow c = 1$

so plane is $x + y - 2z = 1$

Intersects x axis when $y = z = 0$, so $x + 0 - 2 \cdot 0 = 1 \Rightarrow x = 1$

Problem 4 Let L be the line parallel to the planes $x - y + z = 1$ and $2x + y + z = 4 \Rightarrow x = 1$ and that passes through $(1, 3, -3)$. At what point does L intersect the plane $z = 0$?

(a) $(1, -2, 0)$

(b) $(2, 1, 0)$

(c) $(1, 3, 0)$

(d) $(-2, 2, 0)$

(e) $(-1, 4, 0)$

L has direction vector $(1, -1, 1) \times (2, 1, 1) = (-2, 1, 3)$

Passes thru $(1, 3, -3)$

so para. eq. is $\vec{r}(t) = (1 - 2t, 3 + t, -3 + 3t)$

which hits $z = 0$ when $t = 1 \Rightarrow$

$$\vec{r}(1) = (1 - 2, 3 + 1, 0) = (-1, 4, 0)$$