

Quiz 2

MATH 261, CALCULUS III, SPRING 2018

SECTION:

NAME:

Instructions: Solve as many of these problems as you can. Circle the correct answer, and show your work!

Problem 1. Identify the surface $x^2 - y^2 - z = 0$

- (a) hyperboloid of one sheet
- (b) hyperboloid of two sheets
- (c) sphere
- (d) hyperbolic paraboloid
- (e) elliptic paraboloid

Solution: (d) Take traces: $z = 0$ gives $x^2 - y^2 = (x - y)(x + y) = 0$ which is a degenerate hyperbola (an X passing through the origin). All other z give hyperbolas. Freezing x or y gives parabolas. It's a hyperbolic paraboloid.

Problem 2 Which of the following surfaces describes the surface $4x^2 - y^2 - z^2 = 4$

- (a) elliptic paraboloid
- (b) hyperboloid of one sheet
- (c) hyperboloid of two sheets
- (d) cone
- (e) hyperbolic paraboloid

Solution: (c) Take traces: $x = 0$ gives $y^2 + z^2 = -4$ which has no solutions. $x = 1$ gives $y^2 + z^2 = 0$ which is just the point $(0, 0)$. For $|x| > 1$ you get circles of radius $\sqrt{4x^2 - 4}$. Whenever you have circles for most x and nothing for a finite interval of x (i.e. from $(-1, 1)$), that's a hyperboloid of two sheets.

Problem 3 The area of the triangle with vertices at $(a, 0, 0)$, $(0, 2a, 0)$, and $(0, 0, 3a)$ is

- (a) $3a^2/2$
- (b) $5a^2$
- (c) $7a^2/2$
- (d) $6a^3$
- (e) $3a^3$

Solution: (c) Form vectors by picking one of the points as the origin, say $P = (a, 0, 0)$. Call the other two Q and R . Then the area is one half of the length of the cross product $(Q - P) \times (R - P)$.