Quiz 3

SECTION: NAME:

Instructions: Solve as many of these problems as you can. Circle the correct answer, and show your work!

Problem 1. The unit tangent vector to the curve $r(t) = \langle \cos t, \sin 3t, e^t \rangle$ at the point $(1, 0, 1)$ is:

(a) $\langle 0, \frac{3}{\sqrt{10}}, \frac{1}{\sqrt{10}} \rangle$

(b) $\langle 0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle$

(c) $\langle 0, -\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}} \rangle$

(d) $\langle 0, -\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}} \rangle$

(e) $\langle \frac{-\sqrt{3}}{10}, 0, \frac{1}{10} \rangle$

Solution: (a) Find the time t such that the curve passes through $(1, 0, 1)$, i.e. solve $r(t) = (1, 0, 1)$. Solve in one coordinate to save time, $e^t = 1$ means $t = 0$. Then differentiate, $r'(t) = (-\sin t, 3\cos 3t, e^t)$, then evaluate at $t = 0$ to get $r'(0) = (0, 3, 1)$ Find the length: $|r'(0)| = \sqrt{10}$. Then the unit tangent vector is $r'(0)/|r'(0)| = (0, 3, 1)/\sqrt{10}$. A tip: *compute the length only after evaluating at* 0. *It saves time.*

Problem 2. Find the point P on the curve $r(t) = t\vec{i} + t^2\vec{j} + t^3\vec{k}$ at which the tangent vector is parallel to the vector $\langle 2, 4, 6 \rangle$. The coordinates of P are:

(a) $(1, 1, 1)$

(b) $(2, 4, 12)$

(c) $(1, 2, 3)$

(d) $(1, 1, -1)$

(e) $(0, 0, 0)$.

Solution:(a) We're looking for a t such that $r'(t)$ is a scalar multiple of $(2, 4, 6)$, i.e. $(1, 2t, 3t^2) = \lambda(2, 4, 6)$ for some λ. Equating the first coordinate shows $\lambda = 1/2$, then equating the second coordinate shows $t = 1$. So $P = r(1) = (1, 1, 1)$.

Problem 3 Find the equations of the line that passes through the point $(1, 2, 1)$ and that is parallel to the vector tangent to the curve $r(t) = \langle t^2 + 3t + 2, e^t \cos t, \ln(t + 1) \rangle$ at $(2, 1, 0)$.

(a) $x = 1 + 3t,\ y = 2 + t,\ z = 1 + t$

(b) $x = 3 + 2t,\ y = e^t(\cos t - \sin t),\ z = \frac{1}{1+t}$

(c) $x = 1 + 2t,\ y = 2 + t,\ z = 1$

(d) $x = 2 + 3t,\ y = 1 + t,\ z = t$

(e) $x = 2 + 3t$, $y = 1 + 2t$, $z = -3t$.

Problem 4 Suppose the trajectories of two particles are given by $r_1(t) = \langle t+1, 2t^{1/2}, 2^{1/2}t \rangle$ and $r_2(t) = \langle 2t, t^2 + 1, t^2 - 2t + 2^{1/2} + 1 \rangle$. Find the angle between their tangent vectors at their point of collision.

(a) 0

(b) $\pi/6$

(c) $\pi/4$

(d) $\pi/3$

(e) $\pi/2$.

Solution: First find the point of collision, i.e. solve $r_1(t) = r_2(t)$ (note, using a single time variable t means we're looking for a *collision* rather than just an *intersection*). Do this in whatever coordinate is easiest: the first one in this case shows $t = 1$.
The problem is asking for the angle between $r_1'(1)$ and $r_2'(1)$. Find it using the formula $\cos\theta = r_1'(1) \cdot r_2'(1)/|r_1'(1)||r_2'(2)|$.